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The fracture of boron fibre-reinforced 6061 aluminium alloy

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The fracture of 6061 aluminium alloy reinforced with unidirectional and cross-plied 0/90°, 0/90/± 45° boron fibres has been investigated. The results have been described in terms of a critical stress intensity, K_Q. Critical stress intensity factors were obtained by substituting the failure stress and the initial crack length into the appropriate expression for K_Q. Values were obtained that depended on the dimensions of the specimens. It was therefore concluded that, for the size of specimen tested, the values of the critical stress intensity, K_Q, did not reflect any basic materials property.

1. Introduction

There have been a number of attempts to describe the failure of fibre-reinforced materials. The results of some early two-dimensional work of Hedgepeth [1] enabled calculation of the stress concentration factors associated with multiple adjacent broken fibres. Zender and Deaton [2] used Hedgepeth's results to calculate, and experimentally verify, the fracture strength of a reinforced polymer that contained cut Dacron fibres. The importance of matrix plasticity was noted by Hedgepeth and Van Dyke [3] who reported that the elastic stress concentration factors were appreciably reduced when the matrix began to flow.

Three-dimensional stress analysis is extremely difficult and time-consuming to carry out; thus, there have been a number of attempts to describe the failure of fibre-reinforced materials using the macroscopic concepts of linear elastic fracture mechanics. A number of authors have studied reinforced polymeric materials [4–7] and others have studied reinforced metals [8–11]. This type of approach appears to have been reasonably successful despite the fact that very few authors have commented on any size effect that might have influenced the results that they obtained.

In the following paper, experiments are presented in which the strengths of centre-notched and compact tension specimens of 6061 aluminium reinforced with unidirectional and cross-plied 0/90°, 0/90/± 45° boron fibres were obtained. The results indicated that the values of critical stress intensity (K_Q) depended on specimen size, and thus they did not reflect any basic materials property.

2. Experimental technique

2.1. Material specification

The unidirectionally, reinforced material was fabricated by the AVCO Corporation, Lowell, Massachusetts, and the cross-plied material was obtained from DWA Associates, Van Nuys, California. In general, both fabrication processes were similar and consisted of applying a pressure of several thousand psi to foil-filament arrays at a temperature of about 500° C. After fabrication, the material was cooled in air. Individual specimens were cut from the panels using a diamond-impregnated wheel. Notches having a root radius of 12.5×10^{-3} in. were cut using the electric discharge method (EDM).

2.2. Unidirectionally reinforced material

Three groups of centre-notched, 0.10 in. thick specimens were tested. The width (w) of each group was 1.0, 2.0, or 4.0 in., and the ratio of the gauge length to the specimen width was held constant at 3:1. The lengths of the centre notches, $2a$, were varied such that, in each group, a series of specimens were tested having values of $2a/w$ equal to 0.05, 0.10, 0.20, 0.40, and 0.60.

2.3. Cross-plied material

Compact tension specimens were cut from panels in which the fibres were oriented in the 0° and 90° directions. The outer ply was a 0° ply; thus, the thirteen-layer material contained six 90° layers. The dimensions of the compact tension specimens are shown in Fig. 1. The width of the specimen was 2, 4, or 8 in. and the crack lengths were varied such that the effect of five crack length:specimen width ratios could be evaluated. The ratios used corresponded to a/b values of 0.2, 0.4, 0.6, 0.8, and 0.9.

A further series of specimens, similar to those described above, was prepared from material that contained fibres oriented in the 0/90/±45° directions. The layers were arranged in the following sequence: 0/90/±45/−45/0/90/0/−45/±45/90/0. Thus, a balanced laminate was obtained.

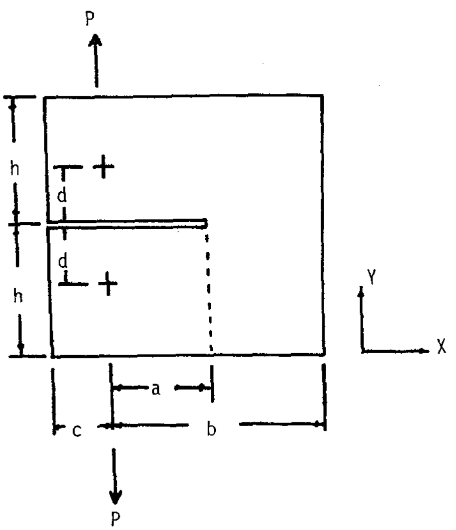

Figure 1 The geometry of the compact tension specimens used in this work. P = load; E = elastic modulus; t = specimen thickness, approximately 0.10 in.; b = 2, 4 or 8 in.; h = 1.2, 2.4, or 4.8 in.; d = 0.55, 1.2 or 2.95 in.

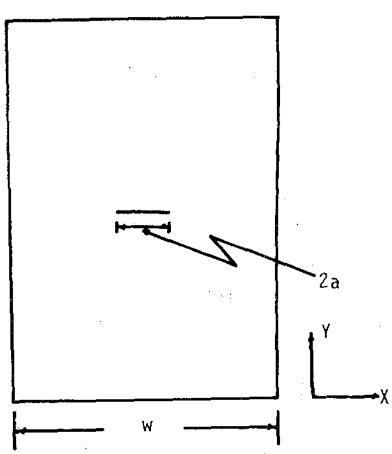

Figure 2 The geometry of the centre-notched specimens used in this work. w = specimen width, 1, 2 or 4 in.; t = specimen thickness, approximately 0.10 in.

The geometry of the centre-notched specimens is shown in Fig. 2. The gauge length of each test piece was approximately 3 in. and the width was 1, 2 or 4 in., respectively. The crack lengths were varied such that the following crack length:specimen width ratios could be studied: $2a/w = 0.05$, 0.10, 0.20, 0.40, and 0.60 for the 1 in. wide specimens and $2a/w = 0.20$ and 0.40 for the 2 and 4 in. wide specimens.

A further series of centre-notched specimens was fabricated from the material which contained boron fibres oriented using the 0/90/±45° sequence mentioned previously. The dimensions of these specimens were identical to those fabricated from the 0/90° material.

2.4. Fibre testing

Individual boron fibres were extracted from selected specimens by dissolving the matrix material in a sodium hydroxide solution. The failure load of each fibre was measured using an Instron screw driven machine. The pneumatic grips of the machine were lined with aluminium to minimize crushing of the fibres and to distribute the load evenly. The gauge length of each fibre was set by using the gauge length adjusting dial of the machine.

2.5. Composite testing

In order to protect the gripped portion and to minimize the tendency to fail in that section, aluminium doubler plates were bonded to the ends of each centre-notched specimen using Eccobond adhesive.

The load was applied to both types of specimen using an hydraulic materials testing system (MTS) operating at a loading rate of 500 lb min^{-1}*. A conventional clip gauge was mounted on the specimen in such a way as to continuously monitor the displacement of either the mid-point of the centre notch or the edge of the notch that was cut into the compact tension specimens.

3. Results

The results obtained by Herring [12], Wright and Iannuzi [13] and Wright and Wills [14] indicate that the strengths of brittle fibres tend to obey a

*1 lb $\text{min}^{-1} \equiv 4.5359 \times 10^{-1}$ kg min^{-1}.

TABLE I The statistical strength data obtained for fibre extracted from composite specimens of different size

Size of original composite		Fibre gauge length (in.)	σ_0 (10^3 psi)	ω	Mean strength (10^3 psi)	Bundle strength (10^3 psi)
width (in.)	length (in.)					
4	24	1	768.7	11.69	473.1	367.6
4	24	2	810.6	10.20	435.2	329.4
4	24	3	877.9	9.23	422.2	314.0
4	24	6	988.3	6.72	328.2	227.7
4	24	12	889.4	6.05	233.9	158.0
2	12	1	874.1	9.64	481.9	364.4

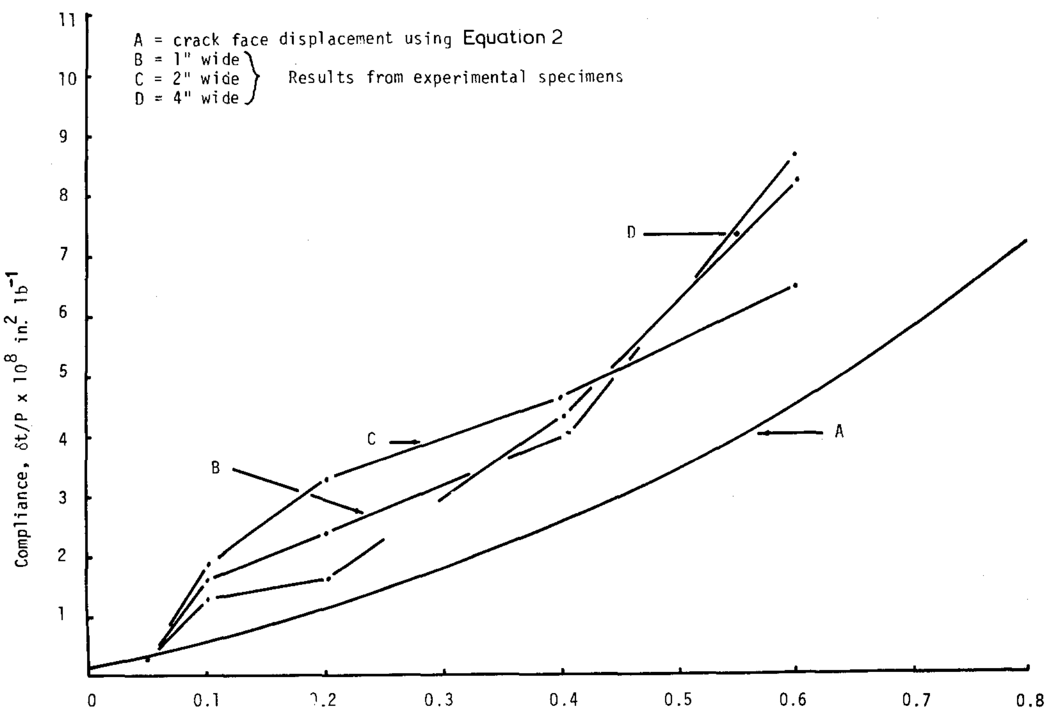

Figure 3 The compliance of various unidirectionally reinforced specimens expressed as a function of crack length to width radio, $2a/w$.

Weibull distribution characterized by

$$G(\sigma) = 1 - \exp\{-(l/D)[(\sigma - \sigma^*)/\sigma_0]\,\omega\} \quad (1)$$

where $G(\sigma)$ is the probability of failure of a fibre subjected to a stress, σ^* the lower limiting strength, assumed here equal to zero, σ_0 the distribution scale factor that reflects the maximum strength that can be obtained from the fibres, ω the distribution shape factor (describes scatter of data), and l/D the fibre length-to-diameter ratio.

Accordingly, the tensile strength data obtained from the fibres tested in this work are presented in Table I, together with the Weibull constants. It is noted that the strengths exhibited by the shorter fibres are larger than those exhibited by the longer ones. In addition, the application of Student's t test at the 95% probability level indicated that identical mean strengths were exhibited by the fibres extracted from the materials prepared by either of the composite suppliers.

3.1. Elastic properties

Elastic compliance curves were obtained by dividing the applied load into the crack opening displacement as computed from the electrical output of the clip gauge. The individual compliance values are shown plotted against the ratio of the crack length:specimen width in Fig. 3 and 4. Included in these graphs are those values of crack face compliance that would be expected from isotropic centre-notched specimens. In this case, the displacement of the crack faces, δ, was calculated using the expressions given by Tada *et al.* [15], i.e.

$$\delta = 4\sigma_{\text{nom}} a/E\, V(a/b) \quad (2)$$

where

$$V(a/b) = 0.071 - 0.535\,(a/b) + 0.169\,(a/b)^2 + 0.120\,(a/b)^3 - 1.071\, a/b \ln(l - a/b).$$

The corresponding expression that was used to describe the compliance of compact tension specimens was:

$$\delta = P/E\, V_1(a/b) \quad (3)$$

where the values of the function $V_1(a/b)$ are taken from the analysis of Roberts [16].

The compliance values of the unidirectional specimens was obtained from the initial part of the load–extension curves. However, compliance values larger than those given by Equation 2 were obtained using this method. Consequently, a small amount of load was reversed during each subsequent test carried out on the cross-plied material and the compliance values were calculated from the slopes of the elastic recovery curves that were obtained. As can be observed, the elastic recovery technique enabled compliance values to be obtained from the pseudo-isotropic cross-plied material that were very similar to those predicted using the isotropic formulations of Tada.

Experimental measurements of the strains generated at or close to the tip of the centre notch present in the unidirectionally reinforced material were carried out using a linear array of transversely oriented strain gauges bonded adjacent to the crack tip. Each gauge, 0.031 in. long, was separated from the preceding one by a distance of 0.024 in. and the length of the total array was 0.25 in.

According to Sih *et al.* [17] the tensile stresses that exist at some reasonable distance from the tip of a long, thin crack in an homogeneous, orthotropic, elastic solid are given by:

$$\sigma_x = K_\text{I}/(2\pi r)^{1/2}(\beta_{22}/\beta_{11})^{1/2} \quad (4)$$

$$\sigma_y = K_\text{I}/(2\pi r)^{1/2} + \sigma_{\text{nom}} \ldots \quad (5)$$

where K_I is the mode I stress intensity factor and

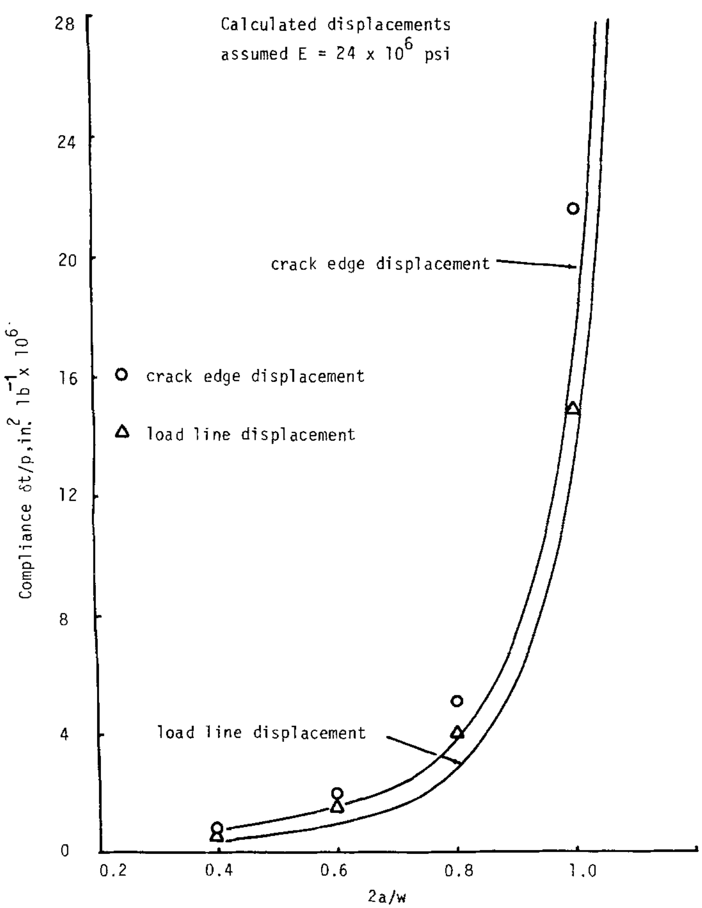

Figure 4 A comparison of the theoretical crack face compliance values with those obtained from experiments carried out on cross-plied 0/90° boron–aluminium.

TABLE II The experimental strains measured across the uncracked ligament compared to those calculated using Equation 7

Gauge number	σ_x calculated using Equation 4 (10^3 psi)	σ_y calculated using Equation 5 (10^3 psi)	ϵ_x calculated using Equation 7 (10^{-6} in. in.$^{-1}$)	ϵ_x measured (10^{-6} in. in.$^{-1}$)
1	12.547	29.229	+487	+150
2	7.288	22.217	+245	+80
3	5.652	20.036	+170	0
4	4.779	18.872	+130	−20
5	4.216	18.121	+104	−50

Applied load = 5000 lb; crack length = 0.2 in.; specimen width = 4 in.

TABLE III The mechanical properties of unnotched and centre-notched specimens of unidirectionally reinforced boron–aluminium specimens

Specimen gauge length (in.)	Specimen width (in.)	$2a/w$	$2a$	Failure load (lb × 10^{-2})	Failure gross (lb)	Stress net (lb)	Failure displacement (in. × 10^4)	$\sigma_3:\sigma_6$	$\sigma_3:\sigma_{12}$	K_Q (10^3 psi in.$^{1/2}$)
3	1	0.00	0.00	19.20	184.90	–	155.8	–	–	–
3	1	0.05	0.05	14.85	139.75	147.11	134.3	–	–	39.2
3	1	0.10	0.10	13.30	128.60	142.89	115.7[2]	–	–	51.3
3	1	0.20	0.20	11.60	110.75	138.44	99.8[2]	–	–	63.6
3	1	0.40	0.40	8.65	83.21	138.68	101.1[2]	–	–	73.3
3	1	0.60	0.60	5.60	53.40	133.50	82.3[2]	–	–	67.6
6	2	0.00	0.00	45.00	215.75	–	491.0[2]	–	–	–
6	2	0.05	0.10	28.20	113.22	140.21	338.2	1.02	–	52.9
6	2	0.10	0.20	21.67	103.04	114.49	248.7	1.21	–	58.1
6	2	0.20	0.40	18.32	87.92	109.90	295.3	1.26	–	71.5
6	2	0.40	0.80	14.17	68.18	113.63	286.0	–	–	85.0
6	2	0.60	1.20	7.29	34.90	81.25	163.0	–	–	62.5
12	4	0.00	0.00	38.80[1]	97.19[1]	–	–	–	–	39.1
12	4	0.05	0.20	28.47	69.68	73.35	341.0	–	1.89	47.7
12	4	0.10	0.40	24.05	59.81	66.46	291.4	–	2.09	55.4
12	4	0.20	0.80	19.60	48.24	60.30	220.0	–	–	57.8
12	4	0.40	1.60	13.37	32.77	54.62	190.1	–	–	60.0
12	4	0.60	2.40	9.55	23.70	59.25	157.8	–	–	–

Failure stresses are mean of two values except where noted by superscript 1. Failure displacements are single values; superscript 2 indicates mean of two values.

σ_x and σ_y are the local stresses in the x and y directions as indicated in Fig. 2. These directions also define the orthotropic axes of the material. Thus, $\beta_{11} = 1/E_x$ and $\beta_{22} = 1/E_y$, where E_x and E_y are the elastic modulus values in the respective directions.

Since for small applied loads the value of σ_y must approach σ_{nom} at some relatively large distance from the crack tip, it follows that a corresponding value for σ_x is given by:

$$\sigma_x = (\sigma_y - \sigma_{\text{nom}})(E_x/E_y)^{1/2}. \tag{6}$$

Use of the elastic constitutive equations enables the value of the transverse strain, ϵ_x, to be calculated for a plane stress specimen

$$\epsilon_x = \sigma_x/E_x - \nu_{yx}\sigma_y/E_y, \tag{7}$$

where ν_{yx} is the major Poissons ratio.

Typical values of the elastic constants of the unidirectional material were found to be $E_y = 32 \times 10^6$ psi*, $E_x = 18 \times 10^5$ psi, $\nu_{yx} = 0.23$. These were used to evaluate Equations 5 to 7. The results of calculations made after assuming that a load of 5000 lb was applied to a 4 in. wide specimen containing a crack 0.2 in. long are included in Table II. It is apparent from these results that the calculated strains are far greater than those measured experimentally. Thus, it can be concluded that since Equation 6 does not predict the correct value of the strains at this loading, then the compliance will not be given by Equation 2 either. This conclusion is not unexpected, for matrix plasticity occurs early in the loading history and this must result in appreciable stress relaxation taking place within the specimen.

*10^3 psi ≡ 220 G Pa.

TABLE IV The failure characteristics of cross-plied boron–reinforced aluminium

Specimen no.	lay/up	Max. load (lb)	Max. stress (psi)	$2a$ (in.)	$2a/w$	w (in.)	t (in.)	K_Q (10^3 psi in.$^{1/2}$)	Net failure stress (psi)
1	0/90	11 000	114 592	0	0	0.997 53	0.096 23	–	–
2	0/90	11 750	120 954			0.997 37	0.097 4	–	–
3	0/90	8 350	88 003	0.05	0.05	0.992 5	0.095 6	24 700	92 635
4	0/90	7 400	76 295	0.1	0.1	0.999 2	0.097 07	30 426	84 772
5	0/90	5 540	57 062	0.2	0.2	1.000 6	0.097 03	32 796	71 327
6	0/90	4 500	47 348	0.4	0.4	0.997 27	0.095 3	34 434	78 913
7	0/90	2 700	27 884	0.6	0.6	0.999 57	0.096 87	35 309	69 710
8	0/90	10 475	108 499			1.003 9	0.096 17		–
9	0/90	8 000	83 989	0.058	0.058	0.997 07	0.095 53	25 404	88 409
10	0/90	7 300	75 616	0.096	0.096	1.000 83	0.096 46	29 532	84 018
11	0/90	5 400	56 047	0.210	0.210	0.999 45	0.096 4	33 095	70 059
12	0/90	3 675	37 959	0.395	0.395	1.002 73	0.096 55	33 149	63 265
13	0/90	2 640	27 422	0.600	0.600	1.002 53	0.096 03	34 724	68 555
14	0/90/±45	9 825	101 801			0.996 3	0.096 87		–
15	0/90/±45	6 850	70 408	0.059	0.059	1.005 07	0.096 8	21 480	74 114
16	0/90/±45	5 800	59 968	0.099	0.099	0.998 13	0.096 9	23 792	66 631
17	0/90/±45	4 780	49 349	0.208	0.208	1.002 7	0.096 6	28 985	61 686
18	0/90/±45	3 400	35 369	0.398	0.398	1.000 3	0.096 1	31 056	58 948
19	0/90/±45	2 425	25 255	0.60	0.60	1.000 20	0.095 83	31 980	63 137
40	0/90	9 000	94 106	0.000	0.000	0.998 3	0.095 8		–
41	0/90	8 100	84 515	0.055	0.055	1.002 2	0.095 63	24 888	88 963
42	0/90	6 900	72 304	0.10	0.10	0.997 9	0.096 5	28 834	80 338
43	0/90	5 650	58 590	0.205	0.20	0.999 3	0.096 5	34 136	73 237
44	0/90	3 800	39 566	0.40	0.40	1.002 2	0.095 83	34 869	65 943
45	0/90	2 450	25 456	0.60	0.60	1.001 2	0.096 13	32 234	63 640
46	0/90/±45	9 700	100 549	0.00	0.00	1.002 5	0.096 23		–
47	0/90/±45 (failed in grips)	6 950	72 384	0.056	0.056	0.996 0	0.096 4	21 510	76 194
48	0/90/±45	5 950	61 721	0.095	0.096	0.992 8	0.097 1	23 976	68 579
49	0/90/±	4 800	49 303	0.205	0.205	1.001 3	0.097 23	28 726	61 629
50	0/90/±45	3 210	33 120	0.395	0.40	1.005 4	0.096 4	28 923	55 200
51	0/90/±45	2 325	24 317	0.595	0.60	1.002 2	0.095 4	30 499	60 792
82	0/90	9 725	50 641	0.40	0.20	1.990	0.096 5	41 058	63 301
83	0/90	9 300	48 346	0.40	0.20	1.975	0.097 4	39 296	60 432
84	0/90	6 775	35 255	0.80	0.40	1.971	0.097 5	43 938	58 758
85	0/90	6 900	35 812	0.80	0.40	1.966	0.098 0	44 634	59 687
86	0/90	17 100	43 951	0.80	0.20	3.930	0.099 0	50 521	54 939
87	0/90	17 090	44 499	0.80	0.20	3.939	0.097 5	51 151	55 624
88	0/90	12 250	32 157	1.60	0.40	3.939	0.097 5	56 678	53 595
89	0/90	31 085	31 085	1.60	0.40	3.939	0.097 0	54 789	51 808

3.2. Fracture properties

In this work, an attempt was made to describe the failure of the composite specimens in terms of a single parameter function by substituting the maximum failure load, P, into one of the following expressions:

Centre-notched specimens

$$K_Q = \{P/wE\}\{\pi a \sec(\pi a/w)\}^{1/2} \quad (8)$$

Compact tension

$$K_Q = \{Pa/bt\}\{29.6 - 185.5\, a/b + 655.7\,(a/b)^2 - 1017.0\,(a/b)^3 + 638.9\,(a/b)^4\}, \quad (9)$$

where K_Q is the critical stress intensity parameter.

Table III contains the failure loads and the associated critical K values that were obtained from unidirectionally reinforced boron–aluminium material. The results were obtained from specimens of three lengths, L, and three widths, w. However, the ratio L/w was maintained constant. It is to be noted that the nominal (gross) failure stresses exhibited by specimens containing identical crack sizes were smaller for the larger specimens. Thus, smaller K_Q values were obtained from larger specimens. This effect was most unexpected since, for metals, a specimen gauge length effect is not considered important.

The failure stresses obtained from cross-plied boron–aluminium specimens are shown in Table IV. In this case, the lengths of the specimens were maintained constant and only the widths were varied. As can be seen, the nominal failure stresses of the specimens that contained identical crack sizes were larger for the wider specimens. Thus, K_Q values increased as the specimen width increased.

3.3. Effect of specimen length
The length effect has been discussed in a recent publication [18] in which it was concluded that, for unidirectionally reinforced composites, the stress concentration effect of the notch becomes constant and independent of notch size providing the notch is larger than some minimum value. It was also concluded that for indentical large cracks, longer specimens were associated with larger stress concentration factors.

It is interesting to compare the conclusions that we have made using the experimentally determined strength values of centre-notched composites with the analytical work of Fichter [19]. Fichter pointed out that the stress concentrating effect of broken reinforcement fibres as calculated by Hedgepeth [1] was obtained by assuming that the fibres were contained in infinitely long and wide specimens. However, if the length of the specimen were finite, as measured by some distance from the notch plane, then Hedgepeth's results should be modified to indicate that the stress-concentrating effect of the notch would not continue to increase but would become constant at some given crack size. Fichter's results are summarized in Fig. 5. As can be observed, for a fibre-reinforced composite of finite length, the stress concentration values, K_r, will not continue to increase as the numbers of broken fibres increase. Rather, a limit to K_r is achieved, the value of which depends on the elastic properties of the composite and the length of the specimen.

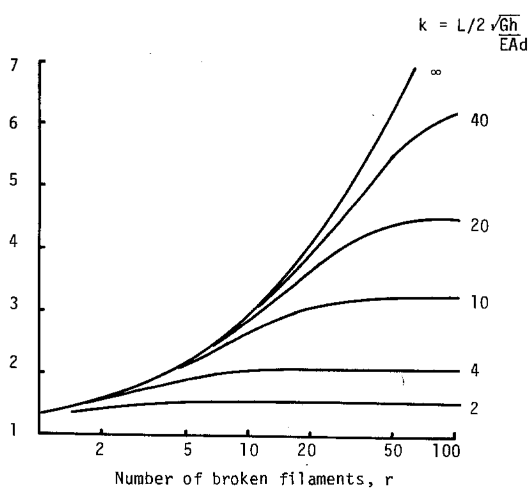

Figure 5 Stress concentration factor for fixed values of k expresses as a function of the number of broken filaments; uniform normal edge displacement.

A quantitative analysis of the experimental results reported herein is possible using Fichter's analysis. For instance, it can be noted that the absolute value of the stress concentration factor, K_r, is a function of the parameter k, where

$$k = L/2(Gh/EAd)^{1/2}.$$

In this expression, $L/2$ is the specimen half length, G is the shear modulus of the matrix, E is the tensile modulus of the fibres of area A, h is the thickness of the specimen, and d is the distance between the fibres. If the value of G is assumed to be about 3×10^4 psi for an aluminium matrix that is deforming plastically, and if h is considered to be about equal to d for a material containing 50 vol% reinforcement, then an approximate calculation indicates that K for a boron–aluminium material is given by

$$k \approx 2.5L.$$

Further inspection of Fig. 5 then indicates that maximum values of K_r will be about 2.7, 3.8 and 5.5 for 3, 6 and 12 in. long specimens, respectively.

Kulkarni [20] has shown, using a finite element analysis technique, that the stress concentrating effect of cut fibres within a unidirectionally reinforced material is a maximum in the first adjacent fibres and then diminishes very rapidly as the distance from the cut fibres increases. Thus, it is reasonable to conclude that the product of the stress concentration factor, K_r, and the net stress at failure will represent the maximum stress in the fibres that are adjacent to the notch. The net failure stresses for unidirectionally reinforced boron–aluminium specimens σ_3, σ_6 and σ_{12} are shown in Table III for the 3, 6 and 12 in. long specimens, respectively. As can be noted, the ratio of the strengths, i.e. $\sigma_3 : \sigma_6 : \sigma_{12}$, is about

TABLE V Calculation of the maximum net stress in first fibres adjacent to a number of fibre breaks

Specimen type	Length (in.)	Width (in.)	Thickness (in.)	Mass net stress at failure (10^3 psi)	Maximum net stress in first fibres adjacent to crack (10^3 psi)
U.	3	1	0.1	140	804
U.	6	2	0.1	115	931
U.	12	4	0.1	57	667
0/90	3	1	0.1	74	794
0/90	3	2	0.1	61	650
0/90	3	4	0.1	54	576

1:1.2:2.0. These values are to be compared with 1:1.4:2.1 that would be predicted using the results of Fichter's analysis.

Additional observations can be made by noting that a mean net stress at failure of the unidirectional material is 140, 115 and 57 x 10^3 psi for the 3, 6 and 12 in. long specimens. Thus, the fibres in specimens containing 47% reinforcement carry respective net stresses of 298, 245 and 121 x 10^3 psi and the first fibres adjacent to the notch will be subjected to maximum stresses of 804, 931 and 667 x 10^3 psi. These values closely approach the value σ_0, i.e. the maximum fibre strength values that can be described by a Weibull distribution. The same calculation can also be applied to the results obtained from the cross-plied material and similar results are obtained; these are shown in Table V.

3.4. Effect of specimen width

The variation in the fracture stress of thin metal specimens can be easily and simply discussed with reference to Fig. 6. In this figure the nominal failure stress is plotted against variations in the crack size and/or specimen width. The curve denoted 1 describes the failure stress of specimens that break when a critical stress intensity is generated at the crack tip. The effect of an increase in specimen thickness is to move the curve downwards and towards the ordinate, as shown in the figure. Essentially, this means that smaller K_Q values are exhibited by thicker specimens. Some thickness is eventually reached beyond which no further movement of the curve occurs; this value of K_Q is then termed K_{IC}. Specimen widths w_1, w_2, w_3, etc., can also be plotted on the horizontal axis of the graph. Straight lines can be drawn

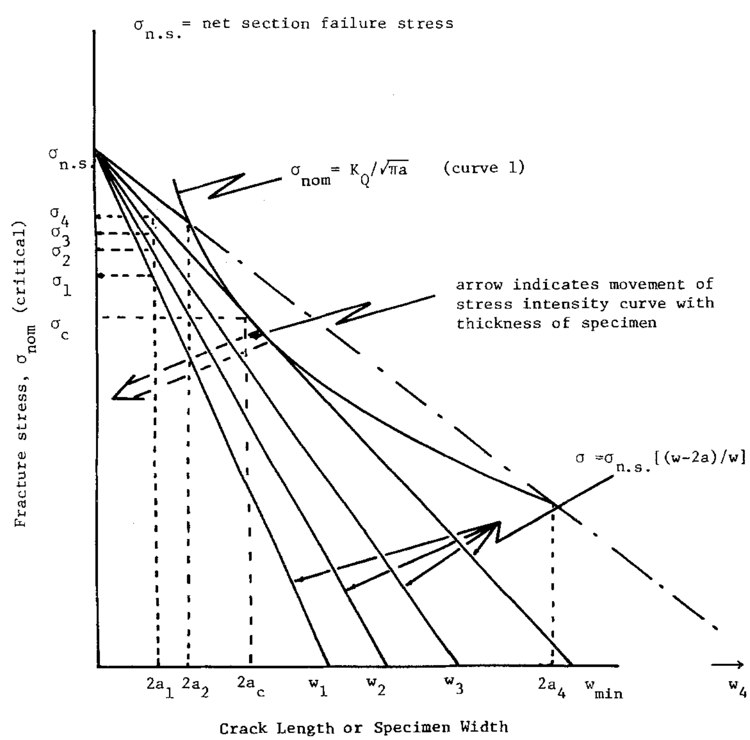

Figure 6 A curve illustrating the tendency of plane stress specimens to fail at a critical net stress.

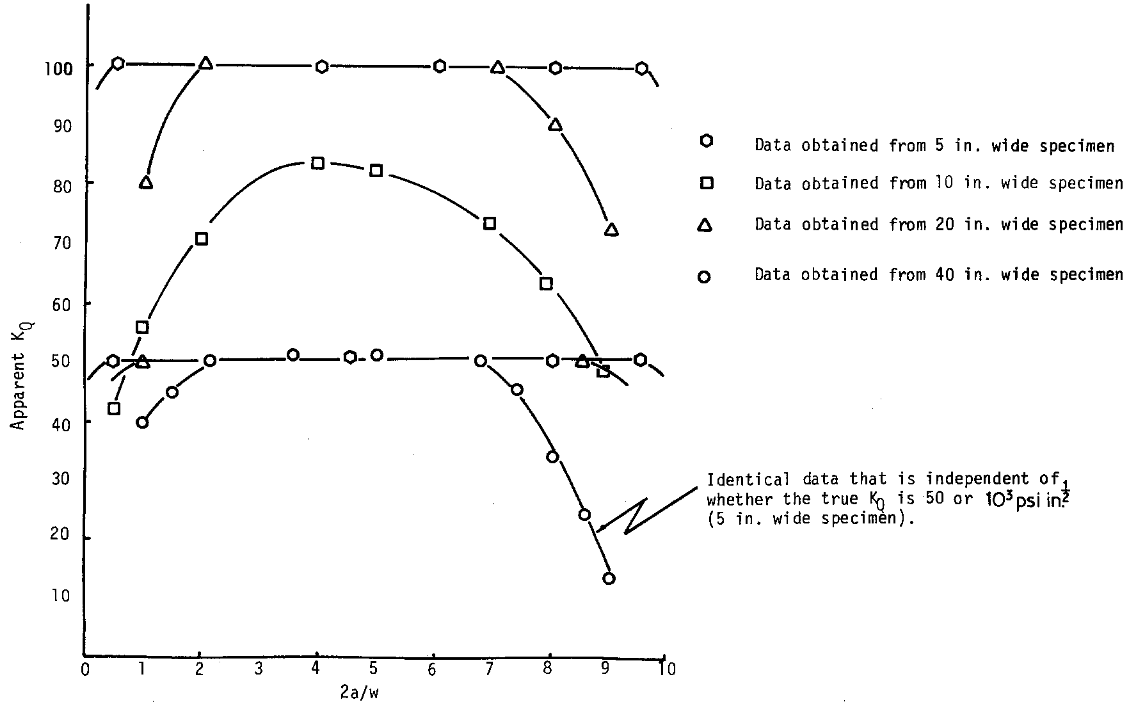

Figure 7 Effect of width on apparent K_Q values (top curves calculated assuming $K_Q = 100 \times 10^3$ psi in.$^{1/2}$; bottom curve calculated assuming $K_Q = 50\,000 \times 10^3$ psi in.$^{1/2}$.

joining the respective specimen widths to a stress which denotes net section failure. These lines then describe the failure stress of notch-insensitive specimens. As can be noted, net section failure will occur in small specimens, i.e. w_1, w_2, w_3, even though a critical stress intensity may describe failure of larger specimens, i.e. w_4. However, even with large specimens, net section failure can occur if the crack sizes are smaller or greater than those denoted in the figure by $2a_2$ and $2a_4$. The values of K_Q that will be obtained from specimens failing in the net section will vary with specimen size and/or crack size. For instance, consider two materials of identical yield stress, 50×10^3 psi, fabricated to a thickness such that real critical stress intensity values of 50 and 100×10^3 psi in$^{1/2}$. would be obtained from ideal experiments. In addition, consider that attempts are made to obtain these real K_Q values using specimens that are 5, 10 20 and 40 in. wide. A curve similar to Fig. 6 can be constructed or a simple analysis can produce results from which the failure stresses of each specimen can be estimated. The results of this type of computation indicate that the actual, experimentally determined K_Q values would vary with the ratio of crack length to specimen width in the manner shown in Fig. 7. It is also apparent from this graph that it would be impossible to obtain true values from 5 or 10 in. wide specimens fabricated from material with a real K_Q of 100×10^3 psi in$^{1/2}$. However, true values could be obtained using all specimens providing a crack length: specimen width ratio of about 0.4 was used. Two effects can be observed: (1) for a given specimen width, the values of K_Q increase to a maximum before decreasing, and (2) large values of K_Q are obtained from larger specimens.

The K_Q values that were obtained from the cross-plied 0/90° 0/90/±45° boron–aluminium composites are shown in Figs. 8 and 9. In this case, since the stress concentrating effect of the notch is independent of notch length, it follows that failure of the composites will occur when the net stress reaches some critical value. Thus, it is not surprising that the shape of these curves is identical to that expected from metal specimens failing by net section yield.

In summarizing the results of the above work, it can be stated that increasing the length of unidirectionally reinforced specimens reduced the K_Q values obtained. Conversely, increasing the width of cross-plied specimens increased the K_Q values obtained. The effect of specimen width was not determined for the unidirectional material,

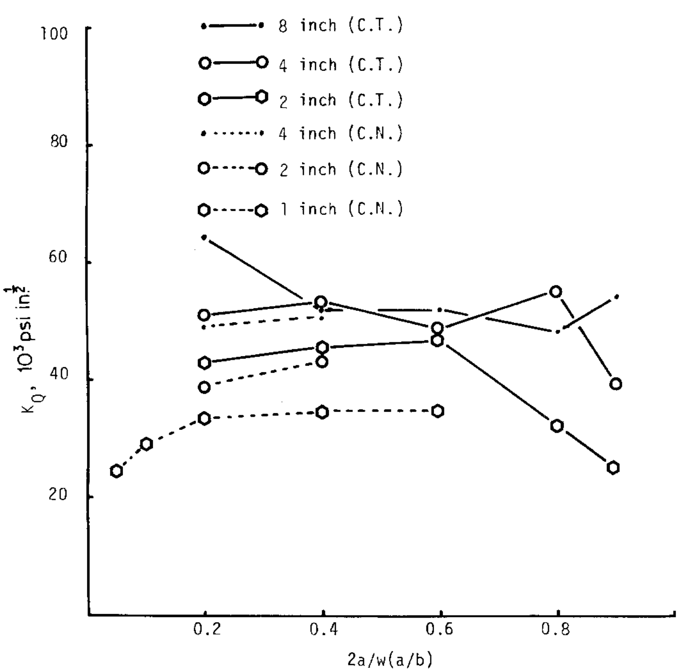

Figure 8 Critical values of the stress intensity factor calculated using the maximum load and the initial crack length; 0/90° reinforcement.

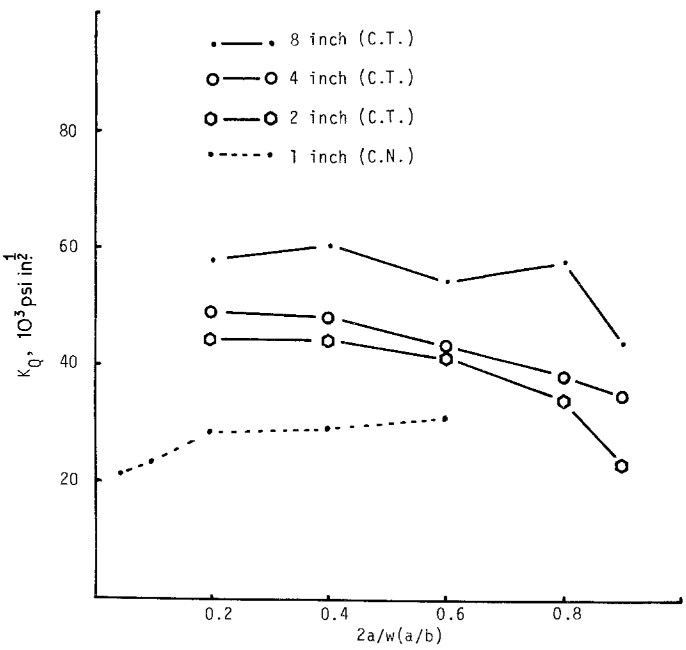

Figure 9 Critical values of the stress intensity factor calculated using the maximum load and the initial crack length; 0/90/± 45° reinforcement.

neither was the effect of specimen length determined for the cross-plied material.

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